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Phase Dynamics at the SQUID and Macro-Realism

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Taking into account some philosophical notions on realism a reformulation of "macro-realism" according to Leggett-Garg is put forward. A macroscopic phase dynamics based on a microscopic SQUID-model is discussed in this context.

1. INTRODUCTION

Macroscopic quantum phenomena (MQP) have been brought to our attention by John Leggett (Leggett and Garg, 1985) in order to remind us of the possibility of their being systems which—as he puts it—are not always in a 'definite macrostate.' He mentions the ring SQUID, which consists of a bulk superconducting ring interrupted by a single Josephson junction. Thus he wants to pose a problem for attempts to solve the measurement problem by appeal to the idea that all macro-observables necessarily lead to superselection rules (breaking of coherent superposition). At the same time he wants to make use of the ring SQUID system in order to refute a certain realistic interpretation of QM, namely, what he calls 'macro-realism.' We would like to take up this latter idea. First, we will translate and discuss it in a different quantum mechanical formalism, namely, in an operator-algebraic frame. By thus providing a more adequate quantum mechanical model, we would like to do more justice to the macroscopic character of the system. Second, we would like to elaborate on the clarity of the relevant general notions involved in the conflict between QM and 'macro-realism.' Most importantly, an improved definition of 'macro-realism' is sought.

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2. REALISM IN PHILOSOPHY

In philosophy, realism is generally conceived of as an *existence* and *independence thesis* (Devitt, 1991; Putnam, 1982; Nola, 1988). Thus understood, it is an *ontological* thesis; it concerns existence or 'being.' More exactly, a realist holds that a certain (kind of) entity *exists* and exists in a *suitably independent* way. So realism needs two qualifications: first, one needs to specify the kind of entity with respect to which one endorses realism; second, one has to specify what counts as 'suitably independent existence.'[2]

The first qualification is either explicitly mentioned in a statement of realism (for example, realism with respect to matter, realism with respect to the future), or the qualification is understood to hold for some unspecified entity (so-called overall realism).

The second qualification is mostly (tacitly) assumed to be understood as 'existence independently of *mind* or *consciousness* ,' which can be viewed as the traditional *philosophical* understanding of realism's independence dimension, expressed by the following definition:[3]

(PLR) Realism (with respect to entity X) $:\Leftrightarrow$ X exists independently of mind or consciousness.

3. REALISM IN PHYSICS

To adapt the definition of realism to physics, it might be useful to change the second qualification into an independence relation that mentions rather observers and measurements than mind or consciousness in general.[4] Thus one gets what might be called the *physical* understanding of realism's independence dimension:[5]

[2] Most prominently, Michael Dummett has proposed an alternative characterization of realism. He takes realism to consist in the assumption of bivalence for a certain class of *statements.* See, for example, Dummett (1963)

[3] For example, Hilary Putnam has defined 'metaphysical realism' as containing (among others) the following thesis: "The world consists of a fixed totality of mind-independent objects" (Putnam, 1982, p. 30). Robert Nola (in a volume containing papers on scientific realism) has defined realism as the position that "there is something which exists in a suitably mind-independent manner" (Nola, 1988, p. 4), More or less equally, Michael Devitt has defined a minimal version of realism ('Weak Realism' or 'Fig-Leaf Realism') as the thesis that "something objectively exists independently of the mental" (Devitt, 1991, Ch. 2.4, pp. 23–25)

[4] There is some discussion of realism in physics which mentions consciousness as well. However, we take it that this is rather a special and 'extreme' way of defining realism in physics.

[5] EPR's 'classical' definition of realism (for a value v of an observable) fits into this physical understanding of the independence relation. For EPR take it to be a sufficient condition of an observable's value v to be real if v can be predicted with certainty (probability equal one). But this sufficient condition reveals that they view an observable's value's existence as real if it exists independent of its measurement or observation. So most probably EPR assume something like what we call the physical understanding of realism's independence relation, namely PSR.

(PSR) Realism (with respect to entity *X*) :⇔ *X* exists independently of (its) measurement or observation.

The first qualification of realism, of course, needs to be mentioned explicitly in any version of realism in physics. For overall realism is not what is at stake so much in physics, and there is no general consensus about what (kind of) entity to which realism in physics holds that might justify leaving this specification implicit. One could even say that what the whole discussion of realism in physics is all about *is* exactly to find out with respect to *which physical entities* one can endorse realism and with respect to which one cannot. So one gets a straightforward definition of ‘macro-realism’ if one fills in for *X* the macroscopic features of physical systems:

(MR) Macro-realism :⇔ *the macroscopic features of physical systems* exist independently of (their) measurement or observation.

Herein, the expression ‘macroscopic features of physical systems’ is meant only as a preliminary notion which would have to be made more precise in order for any critical assessment of MR to be undertaken. We need to become more precise on what counts as ‘macroscopic features.’ And this is exactly what we would like to look into more closely in the following. The strategy will be to take advantage of considering some concrete model in order to gain more precision on general notions like ‘realism’ or ‘macro-....’ We hope that will allow a reasonable specification of MR and, in the end, reasonable judgments on its validity.

4. MACRO-REALISM ACCORDING TO LEGGETT-GARG

John Leggett (together with Anupam Garg) has given a definition of ‘macro-realism’ which consists of the conjunction of two theses, A1 and A2:

(A1) “A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states.”

(A2) “It is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics.” (Leggett and Garg, 1985, p. 857)

Assumption A1 is called ‘macroscopic realism’ by Leggett and Garg. So macroscopic realism’ (A1) is to be distinguished from ‘macro-realism,’ which is the conjunction of A1 and A2. Assumption A2 is called ‘noninvasive measurability at the macroscopic level’.

Leggett’s main thesis is that *macro-realism is incompatible with quantum mechanics.* So he has attempted to construct a conflict between assumptions

A1 and A2 on the one hand, and QM on the other hand. Leggett considers a macroscopic ring SQUID system which consists of a Josephson junction that is superconductively connected into a ring. For this system, he sets out a quantum mechanical description within a two-states-Hilbert-space formalism and derives a quantum mechanical prediction for the temporal correlation $K := K_{12} + K_{23} + K_{34} - K_{14}$, namely

$$K^{(\mathrm{QM})} = 2\sqrt{2} \tag{1}$$

where $K_{ij} := \langle Q(t_i)Q(t_j)\rangle$ and $Q(t)$ is the macroscopic test-observable to be measured (Leggett, 1987). Fundamentally responsible for this result is the occurrence of noncommutation between σ_z and σ_x and between $\sigma_z(t)$ at different times t. For the macro-realistic description he assumes what are essentially the basic ingredients of a deterministic hidden variables theory (for the macro-observables), i.e., a classical conception of an (macro-)observable, and thus derives the macro-realistic prediction

$$K^{(\mathrm{MR})} \leq 2 \tag{2}$$

Two critical comments, however, seem to be reasonable.

1. Leggett's characterization (A1) speaks of (macroscopic) *states* being definite or determinate. However, in the derivation of the MR-prediction Leggett made use of something much stronger. He assumed definiteness of *values* of (macro-) observables. And this was, indeed, necessary in order to derive the result. So the assumption should rather read as:

(A1′) The macro-observables of a macroscopic system will have definite values in all states and at all times.

Now this modified assumption A1′ can be seen to amount to nothing else but *realism with respect to values of macro-observables*, which might be called, for short, 'value-macro-realism':

(VMR) Value-macro-realism :⇔ *definite values of macro-observables* exist independently of (their) measurement or observation.

For the following we will always assume that MR is understood as value-macro-realism (VMR) and, thus, contains A1′ instead of assumption A1.

2. Leggett's derivation of the QM prediction makes use of a two-level formalism. However, this does not reflect anything about the macroscopic nature of the system. So the quantum mechanical description is not really suited for the treatment of *macro*-observables. In this respect, therefore, Leggett's quantum mechanical description is not very satisfactory and one rather would like to have a better adapted description in order to settle foundational questions about the reality of macroscopic quantities. This is

what shall be attempted by means of an operator-algebraic frame in the following.

5. MACRO-REALISM IN OPERATOR-ALGEBRAIC MANY-BODY PHYSICS

In virtue of our previous model discussions, we are able to reconstruct Leggett's considerations within an operator-algebraic framework. The ring SQUID system is described by a model in operator-algebraic quantum theory that is based on a previously elaborated BCS model for each of the superconductors of the ring (Rieckers and Ullrich, 1985a, b; Gerisch and Rieckers, 1996a, b). The SQUID then is described by the tensor product of two such BCS models (Rieckers, 1990). To mention the essential ingredients, one gets in the C*-inductive limit a quasilocal algebra $\mathcal{A} = \mathcal{A}_a \otimes \mathcal{A}_b$ of the single limit algebras $\mathcal{A}_i$ ($i = a, b$). To define a representation of this algebra, one specifies a special state, namely, the limiting Gibbs state ω^β belonging to the finite inverse temperature β. This state allows for a central decomposition of its GNS-triple (π_β, $\mathcal{H}_\beta$, Ω_β) and a direct integral decomposition of the represented von Neumann algebra $\mathcal{M}_\beta := \pi_\beta(\mathcal{A})'' = \pi_\beta(\mathcal{A})^w$ in terms of two macroscopic parameters, one of which we choose as the macroscopic phase angle difference.

The phase difference $\Delta\Theta$ is a central observable of $\mathcal{M}_\beta$ and thus is compatible with all operators of the electronic field algebra. In this frame it is a classical collective observable and expresses an ordered structure which arises by the transition into the superconducting phase: the quantum mechanical phases of the condensed Cooper pairs are locked.

For the description of the experiments the number operators for the electrons, respectively their difference, is crucial. Since ω^β is gauge invariant, these are well-defined, self-adjoint operators in $\mathcal{H}_\beta$. Of the total number operator we pick out a part ΔN which counts the difference of the condensed pairs. Since ΔN commutes with $\pi_\beta(\mathcal{A})$, it represents also a robust collective observable, which, however, is not affiliated with $\mathcal{M}_\beta$. Its spectrum can nevertheless completely be determined by means of notions of $\mathcal{M}_\beta$ (Gerisch and Rieckers, 1996a, b). Thus, we include it into the set of observables and consider the pair of canonically conjugated operators

$$[\Delta\Theta, \Delta N]_- \subset i\pi_\beta(\mathbb{1}) \tag{3}$$

For a rigorous discussion of this commutator see Honegger (1993).

The SQUID is then modeled by a Hamiltonian $\hat{K}$ in $\mathcal{H}_\beta$ which contains the single BCS Hamiltonians, the pair tunneling, and a capacitance term proportional to ΔN^2. Most remarkably, an explicitely solvable phase dynamics can be written as

$$\Delta\Theta(t) := e^{it\hat{K}}\Delta\Theta(0)e^{-it\hat{K}} = \Delta\Theta(0) + B\Delta Nt + \Delta\mu t \tag{4}$$

where $\Delta\mu$ is the applied voltage and B a constant. One finds

$$[\Delta\Theta(t_1), \Delta\Theta(t_2)]_- = iB(t_2 - t_1), \qquad t_1, t_2 \in \mathbb{R} \tag{5}$$

which signifies the quantum character of the dynamical phase difference.

By now, all the crucial ingredients and characteristics that have figured in Leggett's argument are reconstructed in a rigorous operator-algebraic formalism which integrates both the microscopic *and* the macroscopic degrees of freedom. Choosing the phase observables $\Delta\Theta(t_i)$ as Leggett's macroscopic test observables $Q(t_i)$, one could now, in analogy to Leggett's argument, derive a quantum mechanical prediction within the operator-algebraic SQUID model which is in conflict with the MR prediction. Thus, one would arrive at a *violation of temporal Bell-like inequalities by the macroscopic phase observables* $\Delta\Theta(t_i)$.

This violation would show that—if MR is understood as VMR (definite values of macro-observables exist independently of measurement)—MR is contradicted by the operator-algebraic SQUID model. If experiment should confirm the quantum mechanical prediction, MR as VMR would be empirically refuted. Thus, if the more adequate definition of MR as VMR is accepted, our discussion provides good reason to assume that realism is not valid *even in the realm of macroscopic quantum observables.* That does *not* show that all versions of realism are wrong. Rather, it tells us that if one wants to keep a kind of realistic interpretation of QM, one would have to look for some other form of realism which is not problematic in the same way as VMR. The astonishing result is that the same problems for a realistic interpretation of QM as posed by the original Bell-inequalities turn up in the *macroscopic* case, too.

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